Total No. of Questions : 5]

SEAT No. :

P189

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F.Y. B.Sc.
MATHEMATICS
Algebra and Geometry
(Paper - I)(2008 Pattern) (41110)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates:-

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

Q1) Attempt all the subquestions. **[16]**

a) Define - equivalence relation on a set A. An equivalence relation R on Z is defined as, $R = \{(a,b) \in Z \times Z / 2 \text{ divides } a - b\}$. Find the equivalence class of 0 i.e. [0].

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b) Find the values of $\phi(7)$, $\phi(746)$.

c) Express $2 - 2i$ in polar form with principal argument.

d) Find remainder when $x^4 + 16x^3 + 17x^2 + 64x - 100$ is divided by $x - 1$.

e) Find the centre of the conic

$xy - 2x + y - 2 = 0$

f) A straight line makes angles 45° and 60° with the positive x and y axes respectively. Find the angle made by the line with z - axis.

g) Show that the plane $2x - 2y + z + 16 = 0$ touches the sphere $x^2 + y^2 + z^2 + 2x - 4y + 2z - 3 = 0$.

h) Reduce the matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ to it's echelon form. Hence find it's rank.

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Q2) Attempt any <u>four</u> of the following **[16]**

a) Let A = B = C = R. Define $f: A \to B$, $g: B \to C$ by $f(x) = 2x$ and $g(x) = 3x^2 - 1$.
Find formulae for functions *gof* and *fog*.

b) For integers a,b,c if a|bc and gcd (a, b) = 1 then prove that a|c.

c) If $z = 2 + 3i$, find the real and imaginary parts of $\frac{\bar{z}}{z^2}$.

d) Find, 7, 7^{th} roots of 2

e) Find the condition that the equation $x^3 - px^2 + qx - r = 0$ may have three equal roots.

f) Find the remainder when 8^{401} is divided by 13.

Q3) Attempt any <u>two</u> of the following **[16]**

a) State and prove De Moivre's theorem.

b) Find the greatest common divisor of 243 and 198 and find integers *x* and *y* such that gcd (243, 198) = 243*x* + 198*y*.

c) i) Let A = R^2. Let R be the relation on A such that (a, b) R(a_1, b_1) if and only if $a^2 + b^2 = a_1^2 + b_1^2$. Show that R is an equivalence relation. What is [(1, 1)]?

ii) Let $z_1, z_2, z_3 \in C$ such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3|$. Show that $\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$.

d) i) Find the quotient and remainder when $3x^5 - 8x^4 - 5x^3 + 26x^2 - 33x + 26$ is divided by $x^3 - 2x^2 - 4x + 8$.

ii) Solve the equation $x^3 - 5x^2 - 16x + 80 = 0$, the sum of two of it's roots being equal to 0.

Q4) Attempt any <u>four</u> of the following **[16]**

a) The equation *ux* + *vy* becomes $u'x' + v'y'$ after rotating the axes through an angle θ. Prove that $u^2 + v^2 = u'^2 + v'^2$.

b) Derive the normal form of the equation of plane.

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c) Find the points of intersection of the line $\frac{x-8}{4} = y = -(z-1)$ and the sphere $x^2 + y^2 + z^2 - 4x + 6y - 2z + 5 = 0$.

d) Obtain the equation of the circle lying on the sphere $x^2 + y^2 + z^2 - 2x + 2y - 4z + 3 = 0$ and having its centre (2, 2, –3).

e) Find the equation of the plane through the points (2, 3, 2), (4, –5, –3) and parallel to the line $\frac{x-4}{5} = \frac{y+5}{-6} = \frac{z-1}{-2}$.

f) Reduce the matrix $\begin{bmatrix} 1 & -2 & 1 & -4 \\ 1 & 3 & 7 & 2 \\ 1 & -12 & -11 & -16 \end{bmatrix}$ to echelon form. Find it's rank.

Q5) Attempt any <u>two</u> of the following **[16]**

a) Find the angle between the lines whose direction cosines are l_1, m_1,n_1, and l_2, m_2,n_2. Hence obtain the condition for perpendicularity.

b) Reduce the equation $5x^2 + 4xy + 8y^2 - 12x - 12y = 0$ to it's simplest form. Find also eccentricity of the conic.

c) Find λ, if the system of equations

$4x + y + (\lambda^2 - 14)z = \lambda + 2,$

$x + 2y - 3z = 4,$

$3x - y + 5z = 2$

have infinitely many solutions. Find one of them.

d) i) Find the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

ii) Find the equation of a sphere having the circle, $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$; as a great circle.

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