Total No. of Questions : 5]

SEAT No. :

P190

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F.Y. B.Sc.

MATHEMATICS (Calculus)

(Paper - II) (2008 Pattern) (41120)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates:-

1) *All questions are compulsory.*

2) *Figures to the right indicate full marks.*

Q1) Attempt each of the following. **[16]**

a) Prove that $\sqrt{3}+\sqrt{2}$ is an irrational number.

b) Find the solution set of $|x^2| = |x|$.

c) Find the sum of convergent series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

d) Find the smallest positive integer k satisfying $\left|\frac{1}{n} - \frac{1}{n+1}\right| < 0.01$.

e) Find $\lim_{x \to 1+} \frac{|x^2-1|}{x-1}$ if exists.

f) Discuss the continuity of the function

$$f(x) = \frac{x+1}{x^2+3x+2} \text{ in } [1, 2]$$

g) State Rolle's theorem.

h) Evaluate $\lim_{x \to 0} \frac{\log(\sin x)}{\cot x}$

Q2) Attempt any four of the following : **[16]**

a) Prove that $||x| - |y|| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

b) Prove that every convergent sequence of real numbers is bounded. Is the converse true? Justify.

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c) If $x_1 = k > 0$, and $x_{n+1} = \frac{3+2x_n}{2+x_n}, n \geq 1$, then show that the sequence $\{x_n\}_{n=1}^{\infty}$ converges to $\sqrt{3}$.

d) If $x_1 = 2, x_{n+1} = 3 + \frac{1}{2x_n}, n \geq 1$ then show that $\{x_n\}_{n=1}^{\infty}$ is contractive sequence. Find its limit.

e) Show that $\lim_{x \to 1} \left[\frac{x^2 - x + 5|x-1|}{2x - 3|x-1| - 2} \right]$ does not exist

f) Discuss the convergence of the series
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Q3) Attempt any two of the following **[16]**

a) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$.

b) Show that the sequence $\{x_n\}_{n=1}^{\infty}$ where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ is properly divergent.

c) i) If $\lim_{x \to c} f(x)$ exist then prove that $f(x)$ is bounded in a deleted neighbourhood of c.

ii) Show that the series $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ is convergent.

d) i) Find the solution set of $|x| + |x + 1| < 3$.

ii) Show that $\lim_{x \to 0} \frac{1}{x}$ does not exist.

Q4) Attempt any four of the following **[16]**

a) State and prove Cauchy's mean value theorem.

b) Discuss the continuity of the function $f(x)$ at $x = 0$.

where $f(x) = \frac{e^{1/x^2}}{1 - e^{1/x^2}} : x \neq 0$

$= 0 \qquad x = 0$

c) Verify Lagrange mean value theorem for the function $f(x) = \sin x + \cos x$ in $[0, 2\pi]$

d) Find the series expansion of $\log(1 + x + x^2 + x^3)$.

e) Prove that $\frac{x-1}{x} < \log x < x - 1$ for $x > 1$

f) Find 'a' and value of the limit if it exist $\lim_{x \to 0}\left[\frac{a\tan 2x + \tan x}{x^3}\right]$.

Q5) Attempt any two of the following. **[16]**

a) i) If $y = e^{ax}\sin(bx + c)$ then prove that

$$y_n = (a^2 + b^2)^{n/2}\, e^{ax}\sin\left(bx + c + n\tan^{-1}\frac{b}{a}\right).$$

ii) Find 5th derivative of $\cos^2 x$

b) If $y = e^{\tan^{-1} x}$ then prove that

$$(1 + x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$$

c) Show that 'θ' which occurs in the Lagrange mean value theorem tends to limit ½ as $h \to 0$ provided that f' is continuous.

d) Discuss the continuity of the function at $x = 0, 1, 2$,

where $f(x) = \begin{cases} -x^2 & x \le 0 \\ 5x - 4 & 0 < x \le 1 \\ 4x^2 - 3x & 1 < x < 2 \\ 3x + 4 & x \ge 2 \end{cases}$

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